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The Spin Structure of the Effective Quark Hamiltonian and the Hyperfine Splittings of Charmonium*

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ABSTRACT

We provide a simple classical derivation on the spin dependent part of the effective quark Hamiltonian. We suggest that the large S-state hyperfine splitting can be resolved by interpreting that the quark confinement potential is mainly due to an effective scalar exchange.

The observations of monoenergetic γ rays from successive cascades of ψ and ψ' have led to the identification of a series of charmonium states[1,2]; $\chi(3554){:}\,^3P_2$, $\chi(3510){:}\,^3P_1$, $\chi(3413){:}\,^3P_o$, $\eta_c(2830){:}1\ ^1S_o$, $\eta_c' = \chi(3454){:}2\ ^1S_o$, $\psi(3098){:}1\ ^3S_o$ and $\psi'(3684){:}2\ ^3S_o$. While the existence of these states lends considerable support to the charmonium model, it becomes a heavy burden on the existing theories to accommodate the anomalously large ground-state hyperfine splitting $E(\psi)-E(\eta_c) \sim 270$ MeV.[3,4] Recently Schnitzer[5] proposed that such large splitting can be explained by an effective quark-gluon anomalous magnetic moment κ about one quark magneton within the context of a nonrelativistic linear potential model without spoiling the qualitative agreement of the spacings of the P-states. The first two columns of Table 1 summarize the results of Schnitzer's calculation for κ=0 and κ=1.13 (values in parentheses)[5] to be compared with the observed energy spacings.

We wish to point out in this paper that Schnitzer's conclusion is based on an erroneous factor of two in the coefficient of the anomalous magnetic moment contribution to the spin-orbit coupling term. The net result of this correction is to increase the discrepancy of the P-state energy spacings as shown in column 3 of Table 1. (The changes in the S-state hyperfine splitting are probably due to the correction on some numerical errors.) It is clear that the discrepancy becomes too large to be corrected by an effective scalar exchange in addition to the vector exchange without changing the main character of the binding potential. We therefore suggest that a large part of the linear rising binding potential in fact originates from an effective scalar exchange rather than vector exchange. We shall show that the energy spacings can be fitted very well (column 4 of Table 1) by requiring the fraction of scalar exchange to be 88% and κ=5.

Schnitzer's analysis is based on the spin dependent part of the effective quark Hamiltonian derived from the quasi-static limit of the single gluon exchange amplitude to leading order in $(v/c)^2$.[5] However, within such approximation the result can be derived classically in an intuitive and simple manner. The spin-orbit coupling term of the two-body interaction is given by[6]

$$[(1+\kappa_2)(\frac{1}{m_1}+\frac{1}{m_2}) - \frac{1}{2m_2}]\,\frac{1}{m_2 r}\,\frac{dV(r)}{dr}\,\vec{s}_2\cdot\vec{L} + (1\leftrightarrow 2) \quad , \tag{1}$$

where κ, m, r, V, $\vec{s}$ and $\vec{L}$ denote the quark-gluon anomalous magnetic moment, mass, relative distance, quark binding potential and orbital angular momentum respectively, with subscripts labelling the corresponding particle. The first term in the square brackets is the spin-orbit interaction term $-\vec{u}_2\cdot\vec{B}_1 = \frac{1}{m_2}(1+\kappa_2)\vec{s}_2\cdot(\vec{v}\times\vec{\nabla}V)$, with $\vec{r}=\vec{r}_1-\vec{r}_2$ and $\vec{v}=\vec{v}_1-\vec{v}_2$, in the rest frame of particle 2, i.e. the quark-gluon total magnetic moment of particle 2 interacts with the magnetic field generated by the orbital motion of particle 2. The second term in the square brackets corresponds to the Thomas precession contribution, $\frac{1}{2}\vec{s}_2\cdot(\vec{a}_2\times\vec{v}_2)$, originated from the Lorentz transformation from the rest frame of particle 2 to the center of mass system. For the equal mass case, Eq. 1 reduces to

$$\frac{1}{2m^2}(3+4\kappa)\,\frac{1}{r}\,\frac{dV(r)}{dr}\,\vec{s}\cdot\vec{L} \quad ,$$

where $\vec{s}=\vec{s}_1+\vec{s}_2$. There is a discrepancy of 2 in the coefficient of κ compared with the corresponding factor $(3+2\kappa)$ in Eq. 7 of Ref. 5. The correctness of our expression has also been checked by the result from taking the quasi-static limit of the single gluon exchange amplitude.

In order to generalize the spin-orbit term from $\kappa=0$ to the case $\kappa\neq 0$, Jackson[7] erroneously identified the $\frac{1}{m_1}$ term in the square brackets of Eq. 1 with $\kappa=0$ to be the spin-orbit interaction term $-\vec{u}_2\cdot\vec{B}_1$ and the combined term $\frac{1}{2m_2}$ was attributed to be purely quantum mechanical arising from the Pauli reduction of the Dirac matrices. The spin-orbit term thus obtained,

$$\left[\frac{(1+\kappa_2)}{m_1}+\frac{1}{2m_2}\right]\frac{1}{m_2 r}\frac{dV(r)}{dr}\ \vec{s}_1\cdot\vec{L}+(1\leftrightarrow 2)\ ,$$

in the equal mass limit curiously agreed with Schnitzer's result.[5] On the contrary, by proper treatment of relativistic coordinate transformation we clearly demonstrated the classical origin of each term in Eq. 1 without appealing to any quantum mechanical effect.

The spin-spin interaction corresponds to the quark-gluon magnetic moment - magnetic moment interaction. Since relativistic coordinate transformation is not relevant in this case, the classical derivation is quite straightforward. One can follow the electromagnetic derivation step by step except that the 1/r potential is replaced by the binding potential V(r).[8] The result

$$H_{\text{spin-spin}}=\frac{1}{3m_1m_2}(1+\kappa_1)(1+\kappa_2)\ \{2\vec{s}_1\cdot\vec{s}_2\ \nabla^2V(r)$$

$$+\ [\vec{s}_1\cdot\vec{s}_2-3(\vec{s}_1\cdot\hat{r})(\vec{s}_2\cdot\hat{r})]\ \left(\frac{d^2V(r)}{dr^2}-\frac{1}{r}\frac{dV(r)}{dr}\right)\}\qquad(2)$$

agrees with that of Schnitzer.[5]

After the correction of the spin-orbit term, the P-state spacings became too large to be acceptable. It has been suggested that scalar exchange could reduce the strength of the spin orbit interaction.[5] However, any additional mechanism that would reduce the spacings by such large amount is likely to substantially alter the dynamics of the original model. In the following we shall propose a scheme in which scalar exchange is introduced in such a manner that the model of Eichten et al.[9] is straightly maintained. This model is defined by the binding potential

$$U(r) = ar - \frac{\alpha_s}{r} \tag{3}$$

with the parameters $a = 0.149\ \mathrm{GeV}^2$, $\alpha_s = 0.2$ and $m = 1.6$ GeV. While this form of potential is suggestive from the asymptotically free color gluon gauge field theory, it is by no means clear that the effective quark confinement part ar is necessarily originated from vector exchange. We therefore suggest that perhaps only a fraction f of this is contributed by vector exchange and the remaining fraction $1-f$ comes from an effective scalar exchange, i.e. $U(r) = V(r)+S(r)$ with $V(r) = f\,ar - \alpha_s/r$ and $S(r) = (1-f)ar$ denoting potentials due to vector and scalar exchanges respectively.

The contribution of $V(r)$ to the spin dependent part of the Hamiltonian is given as before (Eq. 1 and 2) while the only contribution from the scalar exchange is the Thomas precession term[6]

$$-\frac{1}{2m_2^2 r}\frac{dS(r)}{dr}\vec{s}_2\cdot\vec{L} + (2\to 1)$$

We are now ready to summarize the spin dependent part of the Hamiltonian in this model

$$H_{spin} = [2(1+\kappa) f \frac{a}{r} - \frac{a}{2r} + \frac{3}{2}\alpha_s \frac{1}{r^3}] \frac{1}{m^2} \vec{L}\cdot\vec{s}$$

$$+ \frac{1}{3m^2} [(1+\kappa)^2 f \frac{a}{r} + 3\alpha_s \frac{1}{r^3}] [\vec{s}_1\cdot\vec{s}_2 - 3(\vec{s}_1\cdot\hat{r})(\vec{s}_2\cdot\hat{r})]$$

$$+ \frac{2}{m^2} [2(1+\kappa)^2 f \frac{a}{r} + 4\pi\alpha_s \delta^3(\vec{r})] \vec{s}_1\cdot\vec{s}_2 \qquad (4)$$

Following Schnitzer we have set κ=0 for the contribution to H from the short-distance part of V(r), as expected from the asymptotic freedom of the underlying theory. We obtain the energy differences

$$E(^3P_2)-E(^3P_1) = [\frac{1}{5}(19-\kappa)(1+\kappa)f-1] \frac{a}{m^2} \langle\frac{1}{r}\rangle + \frac{12}{5}\frac{\alpha_s}{m^2}\langle\frac{1}{r^3}\rangle$$

$$E(^3P_1)-E(^3P_o) = \frac{1}{2} [(5+\kappa)(1+\kappa)f-1] \frac{a}{m^2} \langle\frac{1}{r}\rangle + 3\frac{\alpha_s}{m^2}\langle\frac{1}{r^3}\rangle$$

$$E(^3S_1)-E(^1S_o) = \frac{4}{3}(1+\kappa)^2 f \frac{a}{m^2}\langle\frac{1}{r}\rangle + \frac{8\pi\alpha_s}{3m^2}|\phi(0)|^2 \qquad (5)$$

where $\phi(\vec{r})$ is the S-state wavefunction. The matrix elements have been evaluated in Ref. 5; numerically $\frac{a}{m^2}\langle\frac{1}{r}\rangle$ = 25.7 MeV, $\frac{\alpha_s}{m^2}\langle\frac{1}{r^3}\rangle$ = 5.2 MeV for 1P-states, $\frac{a}{m^2}\langle\frac{1}{r}\rangle$ = 41.5 MeV, $4\pi\frac{\alpha_s}{m^2}|\phi(0)|^2$ = 24.3 MeV for 1S-states and $\frac{a}{m^2}\langle\frac{1}{r}\rangle$ = 34.6 MeV, $4\pi\frac{\alpha_s}{m^2}|\phi(0)|^2$ = 30.9 MeV for 2S-states. We find an excellent fit to the observed values for the four energy spacings as shown in Table 1. The values of the two parameters are f = .123 and k=5. If we can take this fit seriously it would indicate that the linearly rising quark confinement potential arises mainly from an effective scalar exchange in contrast to vector exchange. We have yet to find any rigorous reason to support either case. The large value k=5 for the quark-gluon anomalous moment is required to overcome the strong suppression of vector exchange to give the necessary magnetic interaction.

Henriques et al has considered pure scalar exchange for the confinement potential abeit with an exponential damping factor.[10] In spite of a large value of α_s to fit the P-states splitting, the hyperfine splittings remain to be small ($\sim$60MeV).[11]

In summary we have shown that the spin dependent part of the effective quark Hamiltonian to leading order in $(v/c)^2$ can be derived classically. After correcting Schnitzer's errors, we found that the P-state find splittings are too large in comparison with the observed values. However, if we allow the flexibility that the linearly rising potential may arise mainly from effective scalar exchange, we obtain a good fit to the S-state hyperfine splittings and the P-state fine splittings. This possibility has not been entertained previously and it may provide some insight to the dynamics of quark confinement.

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11. They attributed this failure to the possibility of considerable mixing of η_c and η_c' with η and η' , and other effects associated with the two-gluon annihilation process of 0^{-+} mesons. See footnote 2 of Ref. 5 for criticism on this approach.

TABLE 1

Charmonium (hyper) fine splittings in units of MeV. Schnitzer's values are given in parentheses

	$\kappa=0$ $\beta=1$	$\kappa=1.13$ $\beta=1$	$\kappa=5$ $\beta=0.123$	observed
$E(1\ ^3P_2)-E(1\ ^3P_1)$	85	(125)182	40	44 ± 8
$E(1\ ^3P_1)-E(1\ ^3P_o)$	66	(141)170	98	97 ± 8
$E(1\ ^3S_1)-E(1\ ^1S_o)$	72	(300)268	262	268 ± 10
$E(2\ ^3S_1)-E(2\ ^1S_o)$	67	(250)230	225	230 ± 10